CCFU Proof 14

Null Vector: $Q(\varphi, 1) = 0$

Given. Let $\varphi = (1+\sqrt{5})/2$, so $\varphi^2 = \varphi + 1$.

The C_2 characteristic polynomial is $\lambda^2 - \lambda - 1 = 0$. The associated quadratic form (eigenvector convention):

$$Q(x, y) = x^2 - xy - y^2.$$

Companion matrix.

$$A_2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

Eigenvector verification. $v = (\varphi, 1)$ is an eigenvector of A_2 for eigenvalue φ:

$$A_2 \begin{pmatrix} \varphi \\ 1 \end{pmatrix} = \begin{pmatrix} \varphi + 1 \\ \varphi \end{pmatrix} = \begin{pmatrix} \varphi^2 \\ \varphi \end{pmatrix} = \varphi \begin{pmatrix} \varphi \\ 1 \end{pmatrix}. \qquad \blacksquare$$

Convention note. In Proof 9, Q is written as $Q(x, y) = y^2 - xy - x^2$ (state-pair convention). Here $Q(x, y) = x^2 - xy - y^2$ (eigenvector convention). These are coordinate-swapped versions of the same characteristic quadratic structure.

Claim. $Q(\varphi, 1) = 0$.

Proof.

$$Q(\varphi, 1) = \varphi^2 - \varphi \cdot 1 - 1^2 = \varphi^2 - \varphi - 1 = 0, \qquad \blacksquare$$

since $\varphi^2 - \varphi - 1 = 0$ is the characteristic equation of C_2.

Note. The eigenvector $(\varphi, 1)$ of A_2 is a null vector of the quadratic form Q associated with the characteristic polynomial of C_2. This is Q-null, not Minkowski-null. The structural correspondence—that φ is null in its own quadratic form—is an observation, not a physical claim.